POLAR COORDINATES

1. Let O be a fixed point in a plane and $\overrightarrow{OX}$ be a fixed ray in the plane. The fixed point O is called pole and the fixed ray $\overrightarrow{OX}$ is called ***initial line*** or ***polar axis***. Let P be a point in the plane such that OP= r, If $\angle Pox = \theta$ then r, θ are called ***polar coordinates*** of P. The point P is denoted by (r, θ). The nonnegative real number r is called ***radial distance,*** the vector $\bar{r} = \overrightarrow{OP}$ is called ***radius vector*** and the angle θ is called ***vectorial angle*** of the point P.

2. Let P(x, y) be a point in the cartesian coordinate plane. Take the origin O as pole and the positive direction of x-axis as polar axis (initial line). Let (r, θ) be the polar coordinates of P. Then r = OP $= \sqrt{x^2 + y^2}$, $\cos\theta = \frac{x}{r}$, $\sin\theta = \frac{y}{r}$. Thus $x = r\cos\theta$, $y = r\sin\theta$.

3. Conversion of Cartesian coordinates into polar coordinates is $r = \sqrt{x^2 + y^2}$, $\cos\theta = \frac{x}{r}$, $\sin\theta = \frac{y}{r}$.

4. Conversion of polar coordinates into Cartesian coordinates is $x = r\cos\theta$, $y = r\sin\theta$.

5. Let $r, \theta \in R$ and $r > 0$, $0 \le \theta < 2\pi$. Then

   i) $(r, \theta) = (r, 2n\pi + \theta)$ ii) $(-r, \theta) = (r, \pi + \theta)$.

6. The polar coordinates of the origin are $(0, \theta)$ where $\theta \in R$.

7. The distance between the points (r_1, θ_1), (r_2, θ_2) is $\sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_1 - \theta_2)}$.

8. The area of the triangle formed by the points (r_1, θ_1), (r_2, θ_2) (r_3, θ_3) is $\frac{1}{2}|\Sigma r_1r_2\sin(\theta_1 - \theta_2)|$.

9. The points (r_1, θ_1), (r_2, θ_2) (r_3, θ_3) are collinear $\Leftrightarrow$ $\Sigma r_1r_2\sin(\theta_1 - \theta_2) = 0$.

10. If f(x, y) = 0 is the Cartesian equation of a locus S then $f(r\cos\theta, r\sin\theta) = 0$ is the polar equation of S.

11. The polar equation of a line passing through pole and making an angle α with the initial line is $\theta = \alpha$.

12. The polar equation of a line passing through the points (r_1, θ_1), (r_2, θ_2) is $r[r_1\sin(\theta - \theta_1) - r_2\sin(\theta - \theta_2) = r_1r_2\sin(\theta_2 - \theta_1)$.

13. The polar equation of a line passing through the pole and the point (r_1, θ_1) is $\theta = \theta_1$.

14. The polar equation of a line which is at a distance of p from the pole and whose normal makes an angle α with the initial line is $r\cos(\theta - \alpha) = p$.

15. The polar equation of a line parallel to the initial line is $r\sin\theta = p$.

16. The polar equation of a line perpendicular to the initial line is $r \cos \theta = P$.

17. $r \cos (\theta - \alpha) = P \Rightarrow x \cos \alpha + y \sin \alpha = p$ (perpendicular form in Cartesian system)

18. The polar form of the line $ax + by + c = 0$ is $a \cos \theta + b \sin \theta = k/r$ where $k = -c$.

19. The equation $a \cos \theta + b \sin\theta = k/r$ of a line is called general polar equation of a line.

20. The polar equation of a line parallel to the line $a \cos \theta + b \sin\theta = k/r$ is $a \cos \theta + b \sin\theta = k_1/r$.

21. The polar equation of a line perpendicular to the line $a \cos \theta + b \sin\theta = k/r$ is $a\cos\left(\frac{\pi}{2}+\theta\right)+b\sin\left(\frac{\pi}{2}+\theta\right)=\frac{k_1}{r}$.

22. The polar equation of a line perpendicular to the line $r \cos (\theta - \alpha) = p$ is $r \sin (\theta - \alpha) = p_1$.

23. The polar equation of the circle of radius a and having centre at the pole is $r = a$.

24. The polar equation of the circle of radius a and the centre at (c, α) is $r^2 - 2cr \cos (\theta - \alpha) = a^2 - c^2$.

25. The polar equation of the circle of radius 'a' and passing through the pole is $r = 2a \cos (\theta - \alpha)$, where α is vertical angle of centre.

26. The polar equation of the circle of radius 'a' and touching the initial line at the pole is $r = 2a \sin \theta$.

27. The polar equation of a conic in the standard form is $\frac{l}{r}=1+e\cos\theta$.

28. The polar equation of a parabola is $\frac{l}{r}=1+\cos\theta \Rightarrow \frac{l}{r}=2\cos^2\frac{\theta}{2}$.

29. The polar equation of a parabola having latus rectum 4a is $a = r\cos^2\frac{\theta}{2}$